

***ADVANCED CAPITAL
BUDGETING DECISIONS***

CLASS 2-HW-Q5

Topic Decision Tree

Question 5:

Big Oil is wondering whether to drill for oil in Westchester Country. The prospectus is as follows:

Depth of Well Feet	Total Cost Millions of Dollars	Cumulative Probability of Finding Oil	PV of Oil (If found) Millions of Dollars
2,000	4	0.5	10
4,000	5	0.6	9
6,000	6	0.7	8

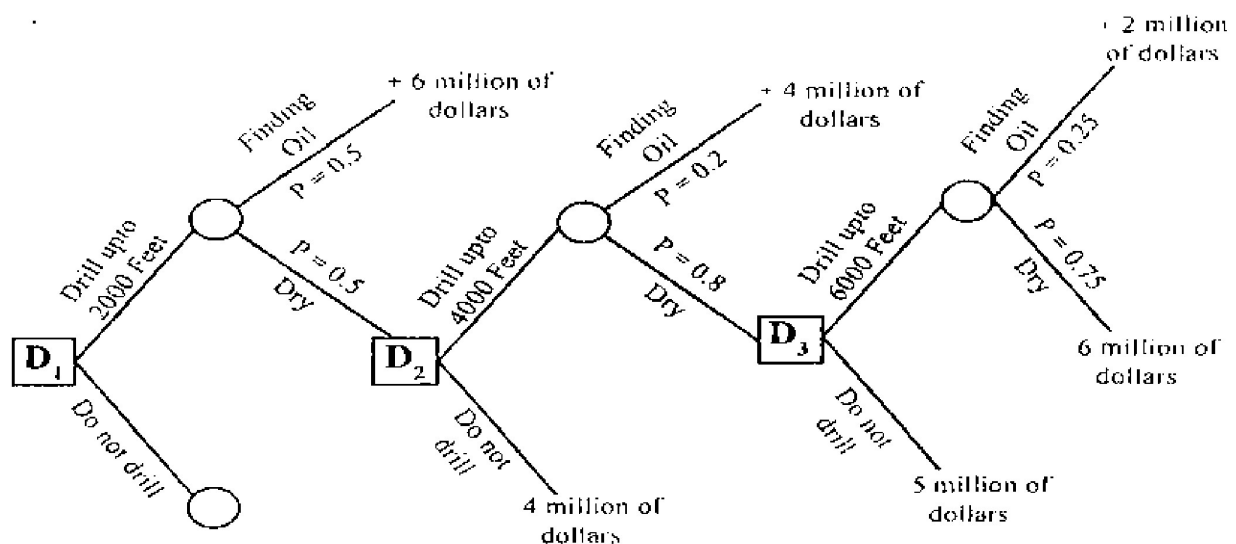
Draw a decision tree showing the successive drilling decisions to be made by Big Oil. How deep should it be prepared to drill?

(Source: ICAI)

ICAI SOLUTION: SLIGHT MISTAKE IN NPV CALCULATION

ANSWER

The given data is easily represented by the following decision tree diagram:



There are three decision points in the tree indicated by D_1 , D_2 and D_3 .

Using rolling back technique, we shall take the decision at decision point D_3 first and then use it to arrive decision at a decisions point D_2 and then use it to arrive decision at a decision point D_1 .

Statement showing the evaluation of decision at Decision Point D_3

	Decision	Event	Probability	P.V. of Oil (if found) (Millions of dollars)	Expected P.V. of Oil (if found) (Millions of dollars)
1	Drill upto 6,000 feet	Finding Oil	0.25	+2	0.50
		Dry (Refer to working note)	0.75	-6	-4.50
					-4.00
2	Do not drill				-5.00

Since the Expected P.V. of Oil (if found) on drilling upto 6,000 feet – 4 millions of dollars is greater than the cost of not drilling – 5 millions of dollars. Therefore, Big Oil should drill upto 6,000 feet.

Statement showing the evaluation of decision at Decision Point D₂

	Decision	Event	Probability	P.V. of Oil (if found) (Millions of dollars)	Expected P.V. of Oil (if found) (Millions of dollars)
1	Drill upto 4,000 feet	Finding Oil	0.2	4	0.80
		Dry (Refer to working note)	0.8	-5	-4.00
					-3.20
2	Do not drill				-4

Since the Expected P.V. of Oil (if found) on drilling upto 4,000 feet – 3.20 millions of dollars is greater than the cost of not drilling – 4 millions of dollars. Therefore, Big Oil should drill upto 4,000 feet.

Statement showing the evaluation of decision at Decision Point D₁

	Decision	Event	Probability	P.V. of Oil (if found) (Millions of dollars)	Expected P.V. of Oil (if found) (Millions of dollars)
1	Drill upto 4,000 feet	Finding Oil	0.5	6	3.00
		Dry (Refer to working note)	0.5	-4.00	-2.00
					1.00
2	Do not drill				NIL

Since the Expected P.V. of Oil (if found) on drilling upto 2,000 feet is 1.00 millions of dollars (positive), Big Oil should drill upto 2,000 feet.

Working Notes:

Let x be the event of not finding oil at 2,000 feet and y be the event of not finding oil at 4,000 feet and z be the event of not finding oil at 6,000 feet.

We know, that, $P(x \cap y) = P(x) \times P(y/x)$

Where, $P(x \cap y)$ is the joint probability of not finding oil at 2,000 feet and 4,000 feet, $P(x)$ is the probability of not finding oil at 2,000 feet and $P(y/x)$ is the probability of not finding oil at 4,000 feet, if the event x has already occurred.

$$\begin{aligned} P(x \cap y) &= 1 - \text{Cumulative probability of finding oil at 4,000 feet} \\ &= 1 - 0.6 \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(x) &= 1 - \text{Probability of finding oil at 2,000 feet} \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$

$$\text{Hence, } P(y/x) = \frac{P(x \cap y)}{P(x)} = \frac{0.4}{0.5} = 0.8$$

Therefore, probability of finding oil between 2,000 feet to 4,000 feet = $1 - 0.8 = 0.2$ we know that

$$P(x \cap y \cap z) = P(x) \times P(y/x) \times P(z/x \cap y)$$

Where $P(x \cap y \cap z)$ is the joint probability of not finding oil at 2,000 feet, 4,000 feet and 6,000 feet, $P(x)$ and $P(y/x)$ are as explained earlier and $P(z/x \cap y)$ is the probability of not finding oil at 6,000 feet if the event x and y has already occurred.

$$\begin{aligned} P(x \cap y \cap z) &= 1 - \text{Cumulative probability of finding oil at 6,000 feet} \\ &= 1 - 0.7 \\ &= 0.3 \end{aligned}$$

$$\begin{aligned} P(z/x \cap y) &= \frac{P(x \cap y \cap z)}{P(x) \times P(y/x)} \\ &= \frac{0.3}{0.5 \times 0.8} \\ &= \frac{0.3}{0.4} \\ &= 0.75 \end{aligned}$$

Therefore, probability of finding oil between 4,000 feet to 6,000 feet = $1 - 0.75 = 0.25$

CORRECT SOLUTION

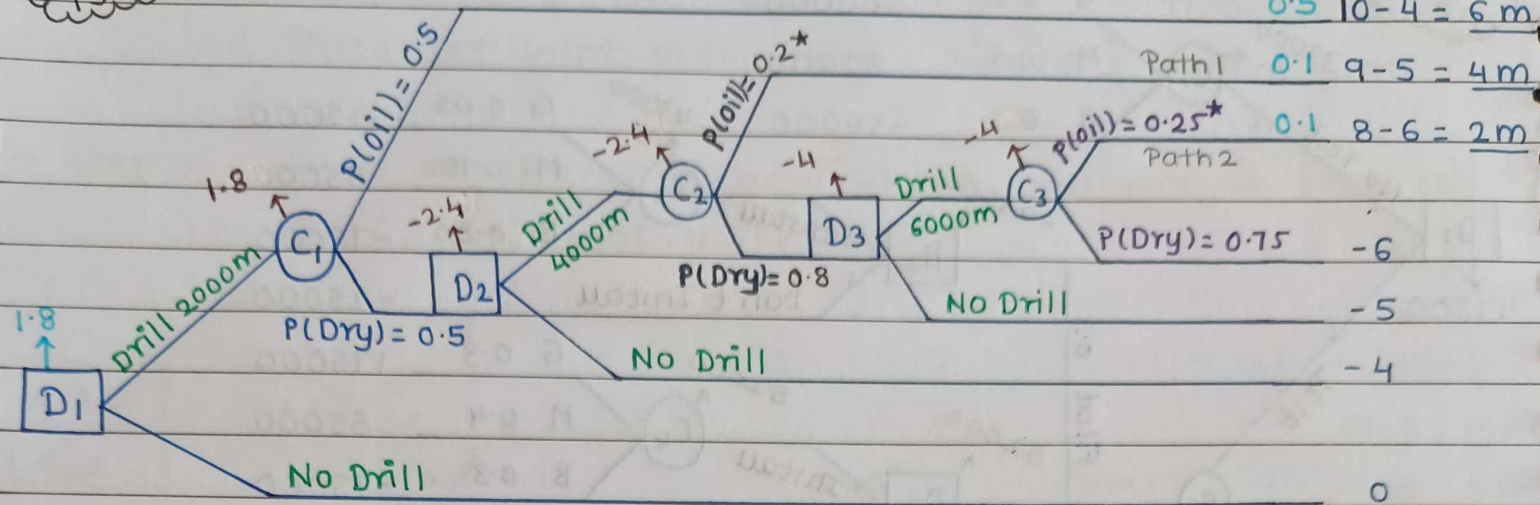
HW. Q.5
Pg.

Joint Probability.

$$0.5 \times 10^{-4} = 5m$$

Path 1 $0.1 \times 9^{-5} = 4m$

Path 2 $0.1 \times 8^{-6} = 2m$



* Joint Prob. of Path 1 = 0.1

• At C3, $EMV = (0.25 \times 2) + (0.75 \times -6)$

$0.5 \times \text{Probability} = 0.1$

$= -4$

$\therefore \text{Prob.} = 0.2$

• At C2, $EMV = (4 \times 0.2) + (0.8 \times -4)$

$= -2.4$

* Joint Prob. of Path 2 = 0.1

$0.5 \times 0.8 \times \text{Prob.} = 0.1$

$\therefore \text{Prob.} =$

• At C1, $EMV = (6 \times 0.5) + (0.5 \times -2.4)$

$= 1.68$

• At D3, $EMV = \text{Max} [-4, -5] = -4$

• At D2, $EMV = \text{Max} [-2.4, -4] = -2.4$

• At D1, $EMV = \text{Max} [1.8, 0] = 1.8$